ON THE PHYSICAL IMPOSSIBILITY OF IDEAL QUANTUM MEASUREMENTS[1]

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Albert and Loewer have argued in this journal [4] that modal interpretations of quantum mechanics are ruled out if the abstract structure of Hilbert space is taken realistically. Their argument contains a dubious inference from a measure-zero set of non-ideal interactions. I look at possible ways to make this inference valid and I conclude that the evidence against the modal interpretation cannot be found in the Hilbert space alone. Instead an analysis of specific cases is required.

Key words: foundations of quantum theory, measurement problem, modal interpretation, non-ideal measurements.

1. ALBERT AND LOEWER'S CRITICISM

Albert and Loewer have presented their argument against the modal interpretation in several forms (see [1-4]), but they all seem to fall under the following general schema:

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(1) Ideal interactions have "measure-zero" in the set of all conceivable quantum interactions.

(2) Measure-zero implies physical impossibility.

(3) Thus, physically possible measurements are not ideal.

(4) But, modal interpretations require measurement interactions to be ideal.

(5) Hence, modal interpretations cannot account for physically possible measurements.

In this general form, the argument seems to conclude that modal interpretations are empirically inadequate. Under a specific, but common, construal of the relationship between theory and phenomena, the argument seems to provide a watertight *refutation* of modal interpretations: the theory has testable observational consequences, these are not borne out in practice, hence the theory must be false.

In this paper I show that this argument by Albert and Loewer cannot refute modal interpretations of quantum mechanics.[2] The problem with the argument concerns premises (1) and (2). These premises are used to derive statement (3), which is then employed as ammunition against the modal interpretation. What would make this part of the argument sound? First, the conclusion (3) must follow from the premises (1 and 2) and second, the premises must be true -of the same states of affairs in the world! More poignantly: "measure-zero" must mean the same in (1) and (2). But not all measures will indicate physical possibility. For instance, it can be shown that the set of states of the composite system that result from a nonideal interaction are nondense in the norm topology of the tensor-product Hilbert space. However there are many well-known counterexamples to the claim that nondensity is a good criterion for physical impossibility. If "measure-zero" means "nondense in some norm topology" then premise (1) is true, but premise (2) is false in general, and hence the overall argument would seem to be unsound.

[2]This conclusion will come as no surprise to some. In fact research on modal interpretations has increased rather than decreased since the advent of Albert and Loewer's criticism!

Albert himself has been trying to put this kind of reasoning at work in classical thermodynamics. The issue there concerns trajectories of thermodynamic states in phase space. Some of these trajectories represent systems with increasing entropy, some with deacreasing entropy. In the neighbourhood of every point in an entropy decreasing trajectory there is an arbitrary large number of points that belong to trajectories that represent entropy-increasing systems. But as Albert himself has pointed out,[3] one cannot immediately draw inferences from this feature of phase space to the likelihood of those trajectories in the real world; instead one needs to investigate the thermodynamic systems, "*case by case*".

Similarly in the quantum domain Albert and Loewer need to produce specific examples of non-ideal interactions to support their conclusion. A measure-zero argument, on its own, will not do the job, -or so I will mantain in this paper. This is not a purely scholastic point: in order to find out what strategies are really needed to save modal interpretations we have to look into the specific cases. We might then choose to give a case by case treatment rather than aiming for a general modal account of non-ideal measurements, an account that will be based upon similarly dubious inferences from measure-zero arguments (some of the work by Bacciagaluppi and Hemmo [5] aims to give such general account).

2. KOCHEN-DIEKS MODAL INTERPRETATIONS

In the Kochen-Dieks modal interpretation interacting quantum sytems are often said to possess two states. The composite is ascribed a *dynamical* state represented by a vector in the tensor product Hilbert space $H_1 \otimes H_2$. The evolution of the dynamical state is governed by the Schrödinger equation. Hence the evolved state is given by the action of a unitary operator on the tensor product Hilbert space. On the other hand subsystems of the composite are also ascribed *value* states, represented by vectors in the corresponding Hilbert spaces H_1 or H_2. The set of possible value states for a subsystem is typically determined by the spectral resolution of the reduced state. The reduced state obtains from the state of the composite by partial tracing and it will normally be a mixed state, i.e., it will be represented by a non-idempotent ($W^2 \neq W$), trace class operator of trace one, a density

[3]see his lecture at the 1995 IUHPS meeting in Florence

operator.

In this paper I deal only with the special case where the composite is in a pure state, but I believe that my arguments also work in the general case where the state of the composite is mixed. If the state of the composite is pure then the spectral resolution of the reduced states will be in terms of the states that figure in the so-called biorthonormal decomposition of the combined state. There is always a biorthonormal decomposition and moreover the decomposition is unique (except for a degenerate combined state). So the subsystems can be ascribed reduced mixed states, represented by density operators on H_1 or H_2, at any instant in time. These facts about the modal interpretation can be summarized in the following diagram:

$$\begin{array}{ccc}
|\Psi\rangle = \sum_i c_i|\psi_i\rangle \otimes |\eta_i\rangle & \xrightarrow{\hat{U}_{1+2}} & |\Psi^t\rangle = \sum_i d_i|\phi_i\rangle \otimes |\xi_i\rangle \\
\downarrow \textit{decomp} & & \downarrow \textit{decomp} \\
W_1 = \sum_i |c_i|^2|\psi_i\rangle\langle\psi_i| & \xrightarrow{?} & W_1^t = \sum_i |d_i|^2|\phi_i\rangle\langle\phi_i| \\
\vdots & & \vdots \\
W_2 = \sum_i |c_i|^2|\eta_i\rangle\langle\eta_i| & \xrightarrow{?} & W_2^t = \sum_i |d_i|^2|\xi_i\rangle\langle\xi_i|
\end{array}$$

The biorthonormal decomposition is unique, so it follows that these mixtures are irreducible representations of the state of the subsystems, and it is tempting to give them an ignorance interpretation. That is, we might be tempted to assume that the subsystems really *possess* one of the pure states in the mixture with the associated probabilities given by the square norms of the coefficients. I am not suggesting that the ignorance interpretation should be given -in fact I'll say something to the contrary in the next section. It is really sufficient for the modal interpretation to provide a way to pick up the pointer position observable that takes values after a measurement interaction and the probabilities for those values. In as much as the modal interpretation does this, and it seems to do it pretty well, it solves the measurement problem without any Kochen-Specker-like contradictions.

3. NON-IDEAL MEASUREMENTS

The modal interpretation is comitted to ideal interactions because it employs the biorthonormal decomposition rule to "pick up" the observables that take values. The different values of the pointer position observable will be perfectly correlated with values of a well-defined observable over the object system; in other words there are no cross terms in the biorthogonal decomposition:

$$|\Psi\rangle = \sum_i d_i|\phi_i\rangle \otimes |\xi_i\rangle \qquad (0.1)$$

But notice that this is a small subset of the set of all possible final states:

$$|\Psi\rangle = \sum_{ij} d_{ij}|\phi_i\rangle \otimes |\xi_j\rangle \qquad (0.2)$$

For most states in this larger set perfect correlations will no longer occur between values of the pointer position observable and any observable over the object system. One can put this in terms of the Hamiltonians that govern the dynamics of the composite. Ideal interaction Hamiltonians will yield final states that are already in the correct biorthonormal form (0.1). For the larger class of Hamiltonians that govern nonideal interactions, the form of the final state of the combined system will be (0.2). Notice that so far the modal interpretation is not really under threat, because modalists could insist that a treatment in terms of ideal interactions can in principle be given to all interesting real cases of physical interaction.

However as Albert and Loewer state [3, p.95]

> ...real measurements are almost never perfect in this sense. In a real measurement there is always some probability of the measuring device making an error.

And in his recent book [1, p.196] Albert concludes that

> If the world is anything less than entirely perfect (and of course it invariably is less than that), then the KDH interpre-tations don't end up doing their job right. And that's that.

So Albert and Loewer's criticism *must* be grounded on the physical impossibility of ideal quantum measurements. To support the claim that ideal measurements are physically impossible, Albert and Loewer have invoked an argument from the measure-zero of the set of Hamiltonians that induce ideal interactions [4, p.301]:

> In the neighborhood of every Hamiltonian that characterizes an ideal measurement, there are Hamiltonians that characterize evolutions like (0.2). In fact, on natural measures the measure of the set of Hamiltonians which correspond to ideal measurements is 0.

However, measure-zero rarely indicates physical impossibility. The classic counterexample is pointing a pointer onto the real line. A natural measure can be laid out over the real line, on the basis of indifference, in such a way that every individual point will have measure-zero, but the whole line will have measure-one. But when the pointer points it will do so on one of those measure-zero points! One can construct highly counterintuitive examples. For instance, the classical mechanical model of the solar system yields a number of possible orbits, given some initial conditions. A measure *could be* laid out over the possible orbits of the planets (in the model), according to which the set of the nine actual orbits will have measure-zero in the set of all possible orbits. And yet nobody will expect to be told that those orbits are impossible, or that they are "*almost never*" realized.

What sort of measure will do the job? And what will the measure be over? There are three possibilities: a measure over Hamiltonians, a measure over composite states, or a measure over the states of the composite.

Consider a measure over the Hamiltonians that govern the evolution of the composite state. It is not clear that the set of Hamiltonians that induce ideal interactions must have measure-zero. Albert and Loewer certainly prove nothing of the sort, and there are in fact substantive reasons to believe that there will be infinite extensions of the partial isometries that induce ideal measurements (some of these reasons are articulated in unpublished work of mine with Gianpiero Cattaneo and Marco Del Seta), and hence a measure based on indifference will not give zero to that priviledged set. But in a sense that's a side issue; even if the set of ideal Hamiltonians was to be given measure-zero, under some measure, the point made previously still holds: physical impossibility does not necessarily follow.

Secondly, consider a measure over the composite state. The advantage of taking the final state of the composite, rather than the Hamiltonians, is that we can now make use of the metrical properties of the tensor-product Hilbert space to define the notion of density. It can be shown that the

set of combined states that result from ideal interactions is non-dense in the topology induced by the norm defined in the space. So in the vicinity of every such point in the topology there are infinite points that represent combined states that result from *non-ideal* interactions. If the metrical properties of the space were to represent real physical distances then it might follow, with some argument, that the states induced by ideal interactions are somehow '*unstable*'. But for similar reasons to the previous ones, it would be a mistake to project the metrical properties of Hilbert space onto the world, unless there is a robust link that connects those instabilities to empirically well confirmed probabilities, such as the quantum probabilities themselves.

This is the third option. For instance one could suppose that the measure over the final states is inversely proportional to the degree of correlation between the reduced states of the subsystems. Although this is contrived it fits Albert and Loewer's intuition well. The states with the larger measure would be those that show complete lack of correlation between the states of the subsystems, i.e. those for which picking every one pure state in the mixture of one subsystem yields an equal distribution over the states in the mixture of the other subsystem. Final composite states that show perfect correlation (the *ideal* case) would then receive measure-zero.

The reason why this measure cannot do the job has to do with the ignorance interpretation of mixtures. Ultimately this measure would be defined over the states of the subsystems. Quantum probabilites are defined as probabilities over the states of the subsystems, and the statistical algorithm gives the probabilities for different quantities in such states. I already indicated that one might be tempted to give an ignorance interpretation to the reduced states of the subsystems because their representation is unique in the modal interpretation. A well-known argument against the ignorance interpretation is precisely that mixtures generally have no unique representation (see for instance [6, p. 144]). For an arbitrary mixed state $W = \sum_i w_i|\psi_i\rangle\langle\psi_i|$ which might be degenerate and where the set $\{|\psi_i\rangle\}$ is not necessarily pairwise orthogonal, there are at least two different representations, as one is always guaranteed by the spectral decomposition theorem. If the mixture is improper then there is no way to tell which representation is physically correct, so we cannot apply ignorance. On the other hand, if we have an independent decomposition rule to fix the correct representation, then we need not worry about nonuniqueness and we can safely apply ignorance (or so the argument would go).

So what are the value states in the modal interpretation? If we give way to temptation then we can assert that the values states are given by the sets $\{|\phi_i\rangle\}$ and $\{|\xi_i\rangle\}$ with the probabilities $|d_i|^2$. And in fact there doesn't seem to be a way to assert that these are the value states, other than to give way to temptation and assume the ignorance interpretation. So the question is: Can we apply the ignorance interpretation? I think that the answer is no. But I think so for a different reason than the one that is sometimes given. The reason is that the so-called "value states" are not really quantum states. In the final section I give a shortened version of an argument to show that there is no consistent ascription of *quantum mechanical* states to the component subsystems, if those subsystems are really interacting.

4. VALUE STATES ARE NOT QUANTUM STATES

If the physics of the particular case at hand indicates that two systems evolve as free particles and have never interacted then nothing is gained in describing the dynamics of the composite. Conversely if the composite is ascribed an entangled state then we must take seriously the fact that the evolution of this composite state will exhibit the results of some physical interaction between the subsystems. One can then take the following criterion as an assumption:

Criterion 0.1 *If two interacting systems are represented as subsystems in a composite system, then the dynamics of the composite state will be given by a unitary operator* $\hat{U}_{1+2}$ *that is not factorizable in terms of independent operators* $\hat{O}_1, \hat{O}_2$ *that govern the evolution of the states of the component subsystems* $\hat{U}_{1+2}|\Psi\rangle \neq \hat{O}_1 \otimes \hat{O}_2|\Psi\rangle$

Note that criterion 0.1 does not apply to systems that have interacted in the past but are currently not interacting -those systems might well show entanglement, while their current Hamiltonian does not factorize. The logical form of the criterion is a conditional where the antecedent requires that the systems are currently interacting. The unitary operator that governs the evolution of this interacting dynamical state will not be factorizable in terms of operators acting independently on the states of the component subsystems. Otherwise we have not taken seriously the physics of the situation.

However the following theorem can be proved ([7] contains a proof and an extension to the general case of mixed combined states):

Theorem 0.1 *If* $W_1^t = \hat{U}_1 W_1 \hat{U}_1^{-1}$, *and* $W_2^t = \hat{U}_2 W_2 \hat{U}_2^{-1}$ *then* $\hat{U}_1$, $\hat{U}_2$ *are unitary operators iff* $\hat{U}_{1+2}|\Psi\rangle = \hat{U}_1 \otimes \hat{U}_2|\Psi\rangle$

In other words it can be shown that the dynamics of the value states is unitary if and only if the unitary operator that governs the evolution of the dynamical state that represents the composite is factorizable[4] in terms of operators acting on the reduced states of the subsystems, H_1 and H_2. But according to the criterion 0.1 the dynamical state does *not* factorize in genuine interactions. It follows that in general there is no unitary evolution for the "value states". Hence no consistent ascription of quantum states can be given to subsystems in interaction. One might want to try to ascribe classical states instead, but classical states have a dynamics of their own, one that will generally be inconsistent with the the Schrödinger evolution of the composite state. The temptation to apply the ignorance interpretation has to be resisted on pain of contradiction.

This argument shows that the label "value *state*" is a misnomer. There are no two kinds of state in the modal interpretation; there is only the dynamical state and that evolves according to the Schrödinger equation. The modal interpretation makes it possible to ascribe values to observables even when the subsystems are not in eigenstates: that is not state-ascription, it's value-ascription.

5. CONCLUSIONS

I have discussed three proposals to lay out a measure that would support Albert and Loewer's argument. They are: measures over Hamiltonians, measures over states of the composite and measures over the ignorantly interpreted reduced states of the components. The first two proposals are non starters, they don't seem to be able to provide an empirical and independently-confirmed measure. The final proposal might initially seem more appealing but in modal interpretations there are no quantum states over which to lay out such measures. A measure-zero argument will not work. The empirical content of Albert and Loewer's argument has to be searched for elsewhere.

Albert has suggested[5] that what underlies the intuition that real mea-

[4] Theorem 0.1 is true up to phase factors of the composite state. Under no circumstances can these phase factors alter the correlations between the subsystems. Of course an interaction with a further measuring device could be set up to reveal these factors, but the theorem would then apply to the subsystem involved in this further interaction.

[5] Private communication, Florence 1995.

surements are never ideal is "common sense". That seems to be right, provided that we understand such "common sense intuition" as the result of accumulated knowledge of instances of real laboratory interactions, and abandon the attempt to ground the intuition directly upon the abstract structure of Hilbert space. The claim that ideal measurements are physically impossible, or very unlikely, can not be justified by appeal to a fallacious inference from a measure-zero set of states in Hilbert space. Such common sense intuition must be, if at all, supported by displaying actual instances of nonideal interactions. In this context, that is work that still has to be done.

In the conclusions to their 1993 paper [4], Albert and Loewer went on to claim that this measure-theoretic inference is warranted by a realist interpretation of quantum theory, and they charged that those who reject the inference are forced to embrace an instrumentalistic view of the quantum theory [4, p.302]:

> We supposed in our original criticisms of the modal interpretation that it was intended as a realist account. But the baneful influences of previous instrumentalistic attitudes can be seen in Bub's and Healy's defenses. [...] This is not much of a defense, since there are also worlds in which non-ideal measurements in our sense are common and which possess outcomes but which the modal interpretation fails to assign outcomes. And, as we argued, it is enormously more likely that our world is one of these worlds.

Is the argument that the Hilbert space provides grounds to believe that among all possible worlds ours (the actual world) is more likely to be one where ideal interactions are scarce? To establish this conclusion would require a great deal of philosophical substantiation in terms of possible world semantics. If on the other hand the purpose is to argue directly for the measure-theoretic inference to the physical impossibility of ideal measurements *in the actual world*, then I believe that this is an argument that won't work. An appeal to scientific realism will not suffice to ground the above mentioned 'common sense intuition' upon the abstract structure of Hilbert space (and hence will not rescue the measure-theoretic inference against ideal measurements). Realism insists that some central terms in mature scientific theories refer. And it insists that some parts of the theory need to be read literally as descriptions of the world. But no (sensible) realist would be unconditionally committed to the existence of all theoretical posits

and entities. In particular those who labour for a realist understanding of quantum mechanics need not be committed to the reification of the metrical properties of Hilbert space.

The realism issue cuts across the present discussion. Realists and instrumentalists alike can avoid the required commitment to the abstract structure of the Hilbert space (required to make Albert and Loewer's argument sound). A simple appeal to the realism/instrumentalism debate will not decide the issue of whether measure-theoretic arguments from Hilbert space provide warrant for the physical impossibility of ideal measurements. It is a debate that will not tilt the balance. There is a long tradition in the history of the quantum theory that teaches caution towards taking the structure of Hilbert space *too* seriously, and that tradition has not always been an altogether instrumentalistic affair!

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