Non-Ideal Measurements and Physical Possibility in Quantum Mechanics

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Abstract

Albert and Loewer have argued against the modal interpretation of non-relativistic quantum mechanics on the basis of its inadequacy in providing accounts of realistic measurements. Their original argument rests on a dubious inference from measure theory. This paper looks at the criticisms levied against their argument and suggests how the argument can be made without appeal to either physical impossibility or measure theoretic arguments.

Albert and Loewer [1, 2, 3] have posed an important difficulty for the modal interpretations advocated by Kochen [12], Healey [10] and Dieks [20] (KHD interpretations). They have argued that KHD interpretations cannot cope with physically realistic measurement situations. One of us (M.S. in [19]) has contested some of the assumptions underlying Albert and Loewer's argument. In this paper we review this and other criticisms of Albert and Loewer's assumptions, and we argue that a sound argument for Albert and Loewer's conclusion against KHD modal interpretations *can* be made, without relying on all of Albert and Loewer's assumptions.

The debate arises out of the need to provide a reliable solution to the problem of quantum measurement. We are looking for a way to model the process of measuring properties of physical quantum systems and a good starting point seems to be to consider two quantum systems, representing the physical entity on which the measurement is to be made and the measurement apparatus, modelled by two Hilbert spaces $\mathcal{H}_{\mathbb{S}}$ and $\mathcal{H}_{\mathbb{M}}$, respectively. These spaces will be tensored together as a standard quantum mechanical composite system, and the initial state of the object-apparatus system will be evolved to a final state after the measurement. What satisfactory conditions will we then impose on the operator which evolves the state and mathematically characterises the measurement?

The most uncontroversial necessary conditions for such an operator $W^{\mathbb{S},\mathbb{M}}$ are the so-called *probability reproducibility condition* and the *calibration condition* respectively (see, for instance, Busch, Lahti and Mittelstaedt [4]):

(1) $\langle\, \varphi | P_{[\varphi_i]} \varphi \,\rangle = \langle\, W^{\mathbb{S},\mathbb{M}}(\varphi \otimes \psi_0) | (I_{\mathbb{S}} \otimes P_{[\psi_i]})(W^{\mathbb{S},\mathbb{M}}(\varphi \otimes \psi_0)) \,\rangle$

(2) $W^{\mathbb{S},\mathbb{M}}(\varphi_i \otimes \psi_0) = \tilde{\varphi}_1 \otimes \psi_1$

for any vector $\varphi \in \mathcal{H}_{\mathbb{S}}$, observable specified by the complete orthonormal set $\{\varphi_i\}$ on $\mathcal{H}_{\mathbb{S}}$ to be measured, initial state $\psi_0 \in \mathcal{H}_{\mathbb{M}}$ of the apparatus, and apparatus observable specified by the complete orthonormal set $\{\psi_i\}$ on $\mathcal{H}_{\mathbb{M}}$. The most general linear operator on $\mathcal{H}_{\mathbb{S}} \otimes \mathcal{H}_{\mathbb{M}}$ satisfying such condition has the form

$$W(\,\cdot\,) := \sum_{i=1}^{n} \langle\, \varphi_i \otimes \psi_0 | (\,\cdot\,) \,\rangle \, (\tilde{\varphi}_i \otimes \psi_i)$$

This operator is a contractive partial isometry on the subspace $\mathcal{H}_{\mathbb{S}} \otimes [\psi_0] := \{\varphi \otimes \alpha\psi_0 : \varphi \in \mathcal{H}_{\mathbb{S}},\, \alpha \in \mathbb{C}\}$ of $\mathcal{H}_{\mathbb{S}} \otimes \mathcal{H}_{\mathbb{M}}$ whose null space is $\mathcal{H}_{\mathbb{S}} \otimes [\psi_0^{\perp}] := \{\varphi \otimes \alpha\psi_0^{\perp} : \varphi \in \mathcal{H}_{\mathbb{S}},\, \alpha \in \mathbb{C}\}$.

The next usual condition is that the operator should be unitary. This leads us to look for unitary completions of the partial isometry W, of which there are, for $\dim \mathcal{H}_{\mathbb{M}} \geq \dim \mathcal{H}_{\mathbb{S}} > 2$, infinitely many. From here onwards we denote by W any such completion of the partial isometry.

At this point the measurement problem can be stated. The operator W maps an initial state of the object-apparatus system to a final state which shows interference between pointer position eigenstates of the apparatus observable. *If* we assume the eigenvalue-eigenstate link for value attribution (EEL), we cannot attribute a definite value to the pointer position of our measuring apparatus and hence have failed to model the real measurement processes of quantum physics, where values are actually measured.

(EEL) states that a system in a pure state φ has a definite value for an observable A if and only if $A(\varphi) = \alpha\varphi$, that is if φ is an eigenstate of the observable A. For an ideal measurement interaction we have that $W(\varphi \otimes \psi_0) = \sum_i \alpha_i \varphi_i \otimes \psi_i$, with φ_i eigenstates of the observable A one is trying to measure on the object system and ψ_i eigenstates of the pointer observable for the apparatus system. Given this overall state for the combined system, the reduced states for the component subsystems are given by two mixtures: $\sum_i |\alpha_i|^2 P_{[\varphi_i]}$ for the object system, and $\sum_i |\alpha_i|^2 P_{[\psi_i]}$ for the measuring apparatus. We could try to give the ignorance interpretation to either mixture, and assume that the state of the system really is one of $P_{[\varphi_i]}$, or the state of the apparatus really one of $P_{[\psi_i]}$. However, this move is well known to contradict the original supposition that the state of the combined system is a superposition, and hence a pure state. For if the reduced states are really pure (and this is what the ignorance interpretation entails) then the state of the combined system is a mixed (non-pure) state, which contradicts our original assumption. So, by reductio, even though the reduced state for the apparatus system is $\sum_i |\alpha_i|^2 P_{[\psi_i]}$, a mixture over eigenstates of the pointer observable, we are not warranted in giving an ignorance interpretation to this mixture and say that the apparatus is definitely in one of the states $P_{[\psi_i]}$. But then we can't appeal to (EEL) and say that the pointer definitely has as value one of the eigenvalues associated with one of the eigenstates ψ_i.

1 The modal interpretation and its criticism

A host of interpretations try to avoid the problem of measurement by renouncing (EEL). Thus, even though the combined system is in a superposition over pointer eigenstates (more precisely eigenstates of the observable $\mathbb{I} \otimes A$), we assume it is possible to assign a definite value to the pointer observable A. These interpretations originate from Everett's work in the 1950's and include many-world interpretations and modal interpretations.

KHD modal interpretations claim that, given a state T, a density operator in the combined system $\mathcal{T}^1_+(\mathcal{H}_\mathbb{S} \otimes \mathcal{H}_\mathbb{M})$, the possible values for the individual systems are selected by the properties picked by the projection operators which diagonalise the reduced density operators in the spaces $\mathcal{T}^1_+(\mathcal{H}_\mathbb{S})$ and $\mathcal{T}^1_+(\mathcal{H}_\mathbb{M})$, obtained by partial trace operations on the state of the combined system.

In particular for every pure state in the composite space $\mathcal{H}_\mathbb{S} \otimes \mathcal{H}_\mathbb{M}$ there is a unique polar decomposition of such state (up to multiplicities), so the basis of this decomposition is formed of vectors from the two subsystems which are themselves pairwise orthogonal. Calculating such decomposition yields the very properties which diagonalise the reduced density operators and thus enables one to "read off" the properties of the two subsystems and the associated values with ease.

Albert and Loewer's argument against modal interpretations has two parts. Firstly, they argue that physically possible measurement interactions are non-ideal. An ideal measurement interaction maps a vector $\varphi_i \otimes \psi_0$ to a special case of the vector $\tilde{\varphi}_i \otimes \psi_i$, namely $\varphi_i \otimes \psi_i$. However, argue Albert and Loewer, it is impossible for measurement devices to record outcomes always in such a perfect way: $\varphi_i \otimes \psi_0$ will be mapped to $\varphi_i \otimes \psi_i$ most of the time, but sometimes our apparatus will misfire, and map the initial state to $\varphi_j \otimes \psi_i$, $j \neq i$. Or more generally the interaction will be such as to yield a final state that is a superposition of pure states of the form $\tilde{\varphi}_i \otimes \psi_i$ such that the $\tilde{\varphi}_i$'s are not pairwise orthogonal.

Secondly, Albert and Loewer argue that modal interpretations cannot cope with non-ideal measurement interactions. If the biorthogonal decomposition algorithm is applied to such final state then the states of the apparatus system will not pick out the right pointer observable. Hence KHD modal interpretations will fail to give values to the pointer position observable in any real measurement.

2 Objections to the Argument

There have been roughly speaking two kinds of responses to Albert and Loewer's objection to the modal interpretation. The overwhelmingly most common response has been to accept the first part of Albert and Loewer's argument, to the effect that ideal interactions are physically impossible, or at least that they are very rare, and to reject the second part of the argument, namely that the modal interpretation cannot cope with non-ideal interactions. Albert and Loewer's latter claim seems to rely upon the assumption that the correct model for imperfect real laboratory measurements is the generalisation of interactions of object system plus measuring apparatus from the ideal to the non-ideal case. However, alternative models of real measurement interactions have been proposed which are not prone to Albert and Loewer's criticisms. These models take the physical environment into account, besides the object system and the apparatus, and consequently expand the tensor product space. The application of the biorthogonal decomposition then becomes a substantive issue, for there are different ways to partition the tensor product space, and the success of the modal interpretation will now depend on the partition that is chosen. As long as these models can be shown to yield empirically indistinguishable predictions as regards measurements of the pointer position observable (and the jury is, arguably, still out on this) the modal interpretation can have a way out. Healey puts the question thus: "What determines whether a particular model of a physical phenomenon within a particular physical theory is appropriate?" [11, p. 51]

Another possible response to Albert and Loewer is to object directly to the first part of their argument. In a previous paper ([19]) one of us has contested the claim that ideal measurements are physically impossible. Why should we think that ideal measurements are physically impossible? We know this, according to Albert and Loewer, because the set of all ideal measurements has measure zero in the set of all possible measurement operators for the measured observable: "on natural measures the measure of the set of Hamiltonians which correspond to ideal measurements is 0" [3, p. 301]. Hence the probability of an ideal measurement taking place is zero.

In particular Albert and Loewer can reason as follows. It turns out that the set I of all unitary measurement operators that yield 'good' modal interactions, namely those for which $\tilde{\varphi}_i \otimes \psi_i$ are such that the $\tilde{\varphi}_i$'s are pairwise orthogonal, forms a set of co-dimension strictly greater than zero in the set of all possible measurement operators. It is then possible to parametrise this set in such a way that the set will need less variable parameters than the whole set of measurement operators. This we can think of in a similar way to the parametrisation of a sphere in $\mathbb{R}^3$ by two angular variables, with the radius variable being held constant. We can then treat the sphere as a set in a cartesian product space of the spaces of all possible angular variables for 'latitude' Θ and 'longitude' Φ, and of all possible radius lengths R, the sphere being a cartesian product $\{r_0\} \times \Phi \times \Theta$ of a singleton set in radius space by Θ and Φ. Provided singleton sets are given measure zero, in the theory of measures on product spaces such sets are known to have measure zero, as the measure of the product set is equal to the (real number) product of the measures on the factor sets: $\mu_{\mathbb{R}^3} = \mu_R \cdot \mu_\Theta \cdot \mu_\Phi$. We assume, for the sake of argument, that the set I of all such unitary measurement operators can be parametrised so that there will be a section of the set consisting of a singleton set. Therefore the set I has measure zero.

However, to argue along this line Albert and Loewer need to make two substantive assumptions, namely that it is *physically appropriate* to treat the set of all possible operators as a product space, with measures defined on all the individual spaces, and that singleton sets on the individual spaces have measure zero. These assumptions are necessary for the measure zero claim to be provably true, rather than true by definition. Without the assumption that the set of all measurement operators is a product space we are not warranted in claiming that the set of ideal measurements has measure zero, as this set, far from being a singleton set, contains infinitely many operators all realising, under current theories of quantum measurement, the same measurement interaction and attribution of measure 0 is a matter of definition. On the other hand we also need the assumption that singleton sets have measure zero in the factor spaces, otherwise our product factorisation of the set will clearly not be sufficient to prove that the measure of the set I of operators is 0.

Now, as Halmos has put it (p 152)

> A measure space might appear as three-dimensional in one context and two-dimensional in another; if for instance, $n = 3$ [the "dimension" of a product space], then we may view X [the whole space on which we define the measure] as $X_1 \times X_2 \times X_3$ or as $X_0 \times X_3$, (where $X_0 = X_1 \times X_2$). [9, p. 152]

Halmos is stressing that the dimensionality of a measure space is not taken to be an intrinsic structural property of the space, but merely serves as a reminder of how we have built the space, and the measure on this space as a product of measures on the factors. In other words, dimensionality is conventional from the measure theoretic point of view.

A probability measure is a totally finite real valued set function μ on a set X such that $\mu(X) = 1$. We can construct a measure on a set X by taking a function $f : \mathcal{P}(X) \to \mathbb{R}$ which is real valued and non-negative, for instance in the following way. Take $X = \mathbb{R}^2$ and define a measure μ on it such that $\mu\{\mathbf{x}\} = \frac{1}{2}\chi_{\{0,1\}} + \frac{k}{2}e^{-x^2} \cdot \chi_{\{x,0\}}$. Here $\mathbf{x}$ is a subset of $\mathbb{R}^2$ (of the appropriate kind), $\chi_{\{0,1\}}$ is the characteristic function over the singleton set $\{0, 1\}$, $\chi_{\{x,0\}}$ is the characteristic function over the real line obtained by setting the y coordinate to be 0 and $\frac{k}{2}e^{-x^2}$ is normalised so that $\mu\{\mathbb{R}^2\} = 1$. This measure is not expressible as a product measure, it might seem very pathological, but it is nevertheless a probability measure on $\mathbb{R}^2$. Presumably whether this is an appropriate measure for $\mathbb{R}^2$ in a physical context, or whether so-called natural measures are physically appropriate, is a matter for empirical investigation.

The topic here has some similarity with discussions of measure theoretic problems in classical and statistical mechanics. Earman, for example, when discussing the problem of singularities of solutions of the equation of motion in Newtonian gravitational theory of point mass particles, remarks that, if all singularities are due to collisions of point particles, a qualified doctrine of determinism might hold. This might seem reinforced by the fact that this is almost always true in the sense that the set of initial conditions which lead in a finite time to collisions is of (Lebesgue) measure zero. But

> There are, however, some caveats about [this]. [. . .] measure zero need not imply either insignificant or ignorable. We would not judge the set of initial conditions giving rise to collisions to be insignificant if, for example, it proved to be dense within the set of states that eventuate in strong interactions (in some appropriate sense) among the particles. Nor would we regard the measure zero set as ignorable if it loomed large within the range of cases we regard, for whatever reason, to be physically interesting. [7].

Or as Sklar has put it,

> any proposal to utilise some mathematical concept, whether measure theoretic or topological, to express the fundamental posits of one's statistical mechanical theory is going to require backing up in the form of a careful exploration of what it is about the physical world, and possibly what it is about the manipulability of that world by nature including the natural things called experimentalists, that makes the mathematical formalism the appropriate one to capture our physical idea [18].

There are, arguably, some differences. Sklar also points out that, in an attempt to fill gaps in measure theoretic arguments for the foundations of statistical mechanics with appeals to topological notions, we use a topology linked to standard distance measures. There is an intuition that this distance measure is "intrinsic" to the basic phase-space representation, while the choice of some measure or other over an algebra of subsets of phase space points is an "imposition". This is presumably because we retain the intuition that phase space points are but a short step beyond the 3-dimensional space in which we live.

The physical intuition about what the appropriate probability measure might be for the set of all possible unitary measurement operators is much more unclear than in discussions of these issues in statistical mechanics, and the connection with realistic geometric representations is harder to establish. The Hilbert space is of course a far more abstract structure, but giving a strongly 'visual' interpretation to the Hilbert space and the operators acting on it yields no automatic justification for adopting a probability measure on the space of operators which will assign measure zero to ideal measurement operators. The justification for the relevant probability measure is problematic, as Sklar's discussions amply exemplify, even in the context of measures over sets of particle trajectories in physical space.

Consider the case of a space representing all the possible orientations of an unbiased die with respect to an observer perpendicularly looking at the die on a plane. We can specify the possible orientations of the die through two angular parameters and identify them with points on a sphere, providing an intuitive picture of this space. What probability measure shall we define for the question as to what orientation will arise when a die is thrown on the plane? I think most of us will agree that on the basis of past physical experience it would make sense to adopt as measure the characteristic function over a set of the obvious six possible orientations, divided by $\frac{1}{6}$. This does not tally up with 'natural' distance measures, but it is still the probability measure that is physically most warranted in this case.

We see then that both assumptions behind Albert and Loewer's appeal to measure theory are questionable. This further strengthens the analysis that Suárez has given of the question [19]. He there claimed that Albert and Loewer's argument, if supported by an appeal to measure-theoretic properties, requires that the Hilbert space structure be interpreted realistically. Indeed it is now clear that even this is not sufficient. For only one particular measure makes sense of Albert and Loewer's claim, and that measure requires justification on its own right.

The second response to Albert and Loewer is then the following: the claim that the set of ideal measurements of an observable has measure zero is an empty claim unless some good *physical warrant* is given for the measure in question. In the absence of such good warrant one can infer measure-zero from probability zero, and one can infer probability zero from physical impossibility, but not vice versa: the inference of physical impossibility from measure-zero is flawed.

To sum up, there are two very different objections to Albert and Loewer's argument, and hence there are two different strategies to restore confidence in modal interpretations. The response that has become standard is to argue that the probability distribution over the measurement observable will typically underdetermine the correct model for an imperfect real measurement interaction; so, different models of any real measurement will be possible; and which model is chosen will be relevant to the success of modal interpretations. Another response points out to the prior fact that we need to back up the claim that real measurement interactions are non-ideal with physics, not measure theory.

3 Physically possible non-ideal measurements

The point that Albert and Loewer make stands, though, as follows: if we can find an example of a real measurement situation which we can all agree is necessarily modelled by a non-ideal measurement operator of a certain kind, then this will pose a problem for the modal interpretation of quantum mechanics. Such an example would constitute a genuinely empirical challenge to modal interpretations. Modal interpretations will not fail *tout court* of course. The example cannot show that ideal measurements are physically impossible, nor even that they are measure-zero. It can only show that KHD modal interpretations fail to ascribe values to the correct observable in the particular physical situation. This section will show that there is at least one such example, namely destructive photon measurements.

These measurements are part of the folklore of philosophy of quantum theory. Henry Margenau evoked them in the context of a famous argument against the projection postulate [15, p. 377]

> . . . perfectly good measurements in atomic physics . . . may "kill" or annihilate a system, and we may be pardoned if we refuse to discuss the eigenstates of nonentities.

A very good example of a destructive measurement was given by Margenau himself, namely the recording of a photon by a photographic plate as a means of measuring its position. Thus, Margenau colourfully claimed [14, p. 30]

> a blackened grain is at once position record and tomb stone of the photon, and [. . .] projection into an eigenstate has certainly not taken place.

In general the detection of a photon's polarisation (which is what must happen if a measurement is to have taken place according to Margenau), by light detectors, photographic plates and the like, result in the absorption of the photon and thus the effective disappearance of it. It will not do to claim that we measure the polarisation of a photon by placing the polariser in the path of a photon. For Margenau this is what we should call preparation; measurement must surely involve detection and, however the two processes of preparation and measurement might be similar, indeed virtually the same in classical mechanics, they certainly aren't the same in quantum physics. The sorts of measurements that Margenau discusses are indeed routine laboratory measurements. On '50's technology they would have worked by having a photomultiplier absorbing the photon, and the resultant energy increase would have triggered the release of an electron. A "chain reaction" mechanism would have led to a signal sufficiently strong to be recorded by a mechanical device increasing the value for number of photon detected by one, on dials pretty similar to a car's mileage counter. By this time the photon would be well and truly dead.

Margenau's objective in discussing destructive measurements was, first of all, to criticise the projection postulate, which he described as follows [1]:

> It is often said that, when a single measurement of an observable Q is made and the result is found to be q, the state is then *known*, and known to be ψ_q, the eigenstate belonging to the measured value.

Margenau noticed that, in the case of destructive measurements, the projection postulate entails the existence of eigenstates of non-entities. If a successful measurement is carried out of an observable Q on a quantum system (perhaps by means of a measurement of the correlated pointer position observable on an interacting measuring device) and a value q of that observable is found, then the projection postulate entails that the state of the quantum system is the corresponding eigenstate. However, if the quantum entity has been destroyed by the act of measurement, this turns out to be the eigenstate of a nonentity; and, as Margenau points out, we should "refuse to discuss the eigenstates of nonentities".

Margenau's argument would seem to cut against us too. We want to argue that destructive measurements are an instance of typical non-ideal laboratory measurements. Thus we would like to show that there is a final state of the composite (and hence an associated reduced state of the system) that exhibits the result of a non-ideal interaction. Consider the case where a photon interacts with an apparatus in the ready state. A measurement interaction is set up—one that obeys the probability reproducibility condition (). The outcome of the measurement is a faithful reading by the apparatus of the values of the observable to be measured on the photon; but it also results in the destruction of the photon. We may characterise a destructive measurement interaction D as the unitary completion of an operator like

$$W(\varphi \otimes \psi_0) = \sum_{i=1}^{n} \alpha_i \varphi_0 \otimes \psi_i = \varphi_0 \otimes \sum_{i=1}^{n} \alpha_i \psi_i \qquad (1)$$

where the initial state of the photon is $\varphi = \sum_{i=1}^{n} \alpha_i \varphi_i$, a superposition over the eigenstates of the required observable Q. (The interaction operator is characterised by the calibration condition mapping every element of the original superposed state of the combined system, $\varphi_i \otimes \psi_0$, into the element of the final state of the combined system $\varphi_0 \otimes \psi_i$.)

Margenau would argue that the final state in equation (1) fails to describe the physical situation, because it does not account for the fact that the measurement has destroyed the photon. The

[1] This argument has been subsequently taken up by Park [16] and Chang [6].

projection postulate would map the above final state to the density operator:

$$\sum_{i=1}^{n} |\alpha_i|^2 P_{[\varphi_0 \otimes \psi_i]}.$$

which enables us to say that "the photon is in state φ_0". The photon however is in no such state, or in any other state, for the photon *exists no more.*

So if Margenau is right, destructive measurements should not really be modelled by (1); and hence they need not constitute an example of measurement interactions that everyone will agree are best modelled as non-ideal. Therefore we must argue, first, that *pace* Margenau, equation (1) correctly models a destructive measurement; and we need to show, second, that the interaction described by (1) is indeed a non-ideal interaction.

Fred Kronz [13] has made clear that the supporter of the projection postulate can contest Margenau's claim. Kronz points out two important things. First, we need not restrict ourselves to think of the projection postulate as a claim that is made of a single system, like a photon or an electron. It is quite possible to state a form of the projection postulate for composite systems: this claims that the non-linear 'projective' transition does not happen at the level of a single system in a measurement situation, but rather at the level of the combined system $\mathcal{H}_{\mathbb{S}} \otimes \mathcal{H}_{\mathbb{M}}$. And what will this 'projective' transition do? It will take the final state of the system as it has evolved through the measurement interaction into a density operator which will precisely satisfy the objectification requirement for the pointer observable, thus allowing an ignorance interpretation of the reduced density operator characterising the pointer system *regardless of what the state of the measured system is at this stage.* So projection here becomes a global phenomenon rather than one characteristic of individual systems.

Such a generalisation of the projection postulate is then coupled by Kronz with the observation that in fact, in the case of destructive measurements, there is no substantive argument to stop us from actually having a no-photon state in the Hilbert space representing an instance of, say, photon absorption by an atom. In other words φ_0 need not represent a state of the photon. But isn't it the case that, in classical mechanics, a state stands for the complete set of values, at a given time, of the dynamic properties of an entity? The entity referred to (usually a point particle) is identifiable independently of the description of the physical situation surrounding it, and of the forces acting upon it.

There is no reason to suppose that exactly the same notion figures in quantum mechanics. A state may represent a contextualised property of the overall physical situation. Perhaps this is a more instrumentalistic rendition of what a 'state' is, in terms of its role in the theory. The state is a formal tool in the prediction of values of certain quantities to be measured in some physical set-up; it is not to be thought of as 'belonging' to a particular entity.

Or, alternatively, one could insist that the same classical notion of state applies in quantum mechanics too, but expand the concept of an 'entity' to cover all kinds of physical set-ups. Such quantum mechanical 'entities' would not be identifiable independently of the description of the overall physical situation; instead, they would be *constituted* by the physical situation. In discussions of holism and non-supervenience of EPR systems, the entangled composite system is often presented as an instance of such quantum mechanical 'entity'.

Margenau must be assuming that the quantum state is, as its classical counterpart, a catalogue of the physical values of properties of an existing entity. It need not be—a quantum state can be said to describe a set of contextual properties of the problem-situation. Indeed in the branch of quantum physics where the photon is treated, namely QED, not only is the absence-of-photons state, the 'vacuum' state, explicitly postulated, it generates some of the more interesting and unusual features of the theory. It appears that we are entitled to discuss the eigenstates of nonentities after all.

For our purposes we are interested in the operator D. This operator most definitely represents a kind of non-ideal measurement as there is only one feasible state that will denote absence-of-photons. The biorthonormal decomposition theorem fails to yield the desired results: if the initial state is $\varphi = \sum_i \alpha_i \varphi_i$, the final state will in fact be $\sum_i \alpha_i \varphi_0 \otimes \psi_i$, which is a factorised state, and the biorthogonal decomposition will pick out properties φ_0 and $\sum_i \alpha_i \psi_i$ as the possessed properties for system and apparatus respectively.

The operator we discuss is formally a special case of a class of operators considered in the literature as possible candidates for measurement operators by Fine amongst others[2] [8].

Contrary to the Albert and Loewer example, this operator does not model a measurement process which is "inefficient" in its recordings, but rather a measurement which is intrinsically non-ideal. And we can 'piggy-back' on Kronz's work: his arguments show that if we want to respond to Margenau's attack against the whole of the quantum theory of measurement we better accept that (1) correctly models destructive measurements. So we have now landed a genuine empirical difficulty for modal interpretations, one which cannot be solved by claims that W models inefficiency of the measurement in the wrong way; such considerations are besides the point here.

Our example of a non-ideal interaction has real force against the first set of responses to Albert and Loewer's criticism. In particular it is worth mentioning that Bub's response to the Albert and Loewer criticism, a variant of the 'wrong model' response, can not work against this example. Bub

[2] Laura Reutsche [17] includes it among her General Unitary Measurement (GUM) operators and uses it to criticise van Fraassen's modal interpretation. These operators fall under the characterisation of measurement evolutions given by Beltrametti, Cassinelli and Lahti [4]. Fine rules out operators such as this as possible measurement operators in the context of measurement with objectification because they do not yield objectification; if objectification is not an issue, however, there seems to be little ground for excluding them.

[5] claims that Albert and Loewer's model of inefficient measurements can be interpreted as such a model only by assuming that the probabilities that the states of the composite system codify are in fact classical probabilities over actual but unknown values for the instrument readings. But this is precisely what we can't assume at the outset when interpreting quantum states in the context of the measurement problem.

However, our example of destructive measurement interactions has no such problem: that the non-ideal destructive measurement is a genuine model of an actual physical phenomenon does not depend on assigning classical probabilities of an instrument making mistakes on the basis of the state's encodings. Furthermore, the final state in the evolution carried through by the destructive measurement is not an entangled state: (1) can be rewritten as $\omega = \varphi_0 \otimes \sum_{i=1}^{n} \alpha_i \psi_i$. Bub's construction of the observables which have actual values for the system in consideration includes the apparatus observable R, characterised by an orthonormal set $\{\psi_i\}$, times the identity operator on the measured system, namely the observable $\mathbb{I} \otimes R$, as well as observables compatible with the projection $P_{[\omega]} \in \mathcal{T}_{+}^{1}(\mathcal{H}_{\mathbb{S}} \otimes \mathcal{H}_{\mathbb{M}})$. The latter, though, can be written as $P_{[\varphi_0]} \otimes P_{[\sum_{i=1}^{n} \alpha_i \psi_i]}$, with $P_{[\varphi_0]} \in \mathcal{T}_{+}^{1}(\mathcal{H}_{\mathbb{S}})$ and $P_{[\sum_{i=1}^{n} \alpha_i \psi_i]} \in \mathcal{T}_{+}^{1}(\mathcal{H}_{\mathbb{M}})$. This implies that the projector $P_{[\sum_{i=1}^{n} \alpha_i \psi_i]}$ will guarantee the presence in the partial algebra of observables which have actual values of observables of the form $\mathbb{I} \otimes O$, M-observables in Bub's own terminology, which will be incompatible with the observable $\mathbb{I} \otimes R$, given that the projection $P_{[\sum_{i=1}^{n} \alpha_i \psi_i]}$ is not orthogonal to the ψ_i's, the eigenstates of the observable R.

Summing up, destructive measurements seem to satisfy the following desiderata:

- They are actual, fairly common measurements in quantum physics;
- They can be modelled within the traditional framework for quantum measurements;
- They are essentially non-ideal, in that the final states of the measured system are not pairwise orthogonal—in fact, if the process is to be modelled in a satisfactory manner, they must be the same state.

Of course, it is of no use to reappropriate Albert and Loewer's measure-theoretic arguments to deny plausibility for this kind of measurements. It certainly is the case that the co-dimensionality of unitary operators satisfying a condition such as 1, on page 9, is larger than that of ideal operators, so that there are even 'fewer' of them. But this, as explained before, is besides the point. Just as measure theoretic arguments cannot be used to substantiate Albert and Loewer's critique, nor can they be used in the opposite direction, to undermine Albert and Loewer's critique. Destructive measurements *happen*, and hence they are physically possible; whether these measurements are assigned probability zero or non-zero by some particular measure is simply irrelevant to the fact that they are real, physical, laboratory measurement interactions.

4 Conclusions

Destructive measurements must then fall outside the domain of modal interpretations of quantum theory, which indicates that these interpretations do not possess full generality. All this is of course not a claim for ruling out modal interpretations in general. We already established that no general refutation can be forthcoming. Indeed, in reviewing the recent literature springing from Albert and Loewer's original proposal, one may be forgiven for wondering what the fuss has been all about. In the absence of a detailed discussion of concrete examples of real, physical, non-ideal interactions, Albert and Loewer's criticism is rather empty, and can have no bite against any interpretation of quantum theory.

We have tried to redress the balance by pointing out a family of real laboratory interactions that seem to be necessarily non-ideal in the required sense. We don't claim to have the definitive analysis of destructive measurement interactions, but we hope to be focusing attention on what really matters. Albert and Loewer's argument against the modal interpretation is an empirical, not a mathematical, argument—and hence it must be discussed at the right level of empirical detail.

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