

Scientific representation: against similarity and isomorphism

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Abstract *I argue against theories that attempt to reduce scientific representation to similarity or isomorphism. These reductive theories aim to radically naturalize the notion of representation, since they treat scientist's purposes and intentions as non-essential to representation. I distinguish between the means and the constituents of representation, and I argue that similarity and isomorphism are common but not universal means of representation. I then present four other arguments to show that similarity and isomorphism are not the constituents of scientific representation. I finish by looking at the prospects for weakened versions of these theories, and I argue that only those that abandon the aim to radically naturalize scientific representation are likely to be successful.*

1. Theories of scientific representation

Many philosophers of science would agree that a primary aim of science is to represent the world (van Fraassen, 1981, 1987; Friedman, 1982, Chap. 6; Kitcher, 1983; Giere, 1988, 1999a; Morrison & Morgan, 1999; Cartwright, 1999; Morrison, 2001, Chap. 2; a well-known dissenter is Hacking, 1983). What those philosophers understand by “represent” is, however, a lot less clear. No account of representation in science is well established. Perhaps this is not surprising. Consider the following four very different examples of successful scientific representation, drawn from engineering and mathematical physics, respectively: a toy model of a bridge; an engineer's plan for a bridge, such as the Forth Rail Bridge—an example carefully documented by Michael Baxandall (1985, Chap. 1); the billiard-ball model of gases (Hesse, 1967); and the quantum state diffusion equation for a particle subject to a localization measurement (Percival, 1999). What could there be in common between such disparate models that allows them to represent?

I choose these examples mainly because they illustrate the range and variety of representational devices in science. In these examples we may usefully distinguish between the *source* and the *target* of the representation. Roughly, the source is the vehicle of the representation, the target is its object. In the first two examples the source is a concrete physical object and so is the target. In the third example we may describe the source as a physical system and the target as a state of a nature. In the fourth example the target is a physical phenomenon and the source a mathematical entity, an equation.

In all these cases A is the source and B is the target when and only when “A represents B” is true.

There are of course many other kinds of representational media in science. The sources of scientific representations may be concrete physical objects, systems, models, diagrams, images or equations; and similarly for possible targets. The only thing that they have in common is that they all putatively include real entities in the world; and there does not seem to be any property in particular that allows any of them to perform one or the other function.

I take it that a *substantive* theory of scientific representation ought to provide us with necessary and sufficient conditions for a source to represent a target. It is natural to expect these conditions to agree with our underlying intuitions about ordinary representation in general; but we should not necessarily require the conditions of scientific representation to be identical to those for ordinary representation. Neither should we require that a theory of scientific representation be able to explain how humans have evolved the capacity to generate representations, or mental images of the world; although this is an independently interesting issue (see, e.g. Woodfield, 1991).

In addition, a good theory may provide us with insight into some of the features that are normally associated with scientific representations such as accuracy, reliability, truth, empirical adequacy, explanatory power; but again we shall not assume that this is a requirement. In other words, we shall not require a theory of representation to mark or explain the distinction between accurate and inaccurate representation, or between a reliable and unreliable one, but merely between something that is a representation and something that is not.¹ This presupposes a distinction between the conditions for x to be a representation of y , and the conditions for x to be an *accurate* or *true* representation of y . Both are important issues, but they must be addressed and resolved separately. Science often succeeds at constructing representations of phenomena, but it rarely succeeds at constructing completely accurate ones (see, e.g. Bailer-Jones, 2003). On discovering particular inaccuracies in the representation we are very rarely inclined to withdraw the claim that it is *a* representation. Thus a graph can be a more or less accurate representation of a bridge, and a quantum state diffusion equation can be a more or less accurate representation of a particular instance of localization. I have here little to say about what makes one representation more accurate than another.²

In this article, I critically discuss two proposals for a substantive theory of scientific representation along these lines. The intuition underlying these theories may at first appear natural and pervasive, but I will argue that on careful analysis it must be resisted. The intuition is that a source A is a representation of a target B if and only if A, or some of its parts or properties, constitute a mirror image of B, or some of its parts or properties. A and B are entities occurring in the world as described by science, so a thorough scientific investigation of all the facts about A and B and their relation should thus suffice to settle the matter. This is perhaps best summarized by means of a slogan: “scientific representation is a factual relation between entities in the world that can be studied by science”. Since the relation of representation is factual it cannot involve essential or irreducible judgements on the part of agents.

One sense in which we may naturalize a concept is by reducing it to facts, and thus showing how it does not in any essential way depend upon agent’s purposes or value judgements (Putnam, 2002; van Fraassen, 2002). The two theories that I criticize here are naturalistic in this sense, since whether or not representation obtains depends on facts about the world and does not in any way answer to the personal purposes, views or interests of enquirers. These theories have the virtue of guaranteeing the objectivity

of scientific representation which, unlike linguistic representation perhaps, is certainly not a matter of arbitrary stipulation by an agent.

However, other non-naturalistic conceptions may guarantee the appropriate level of objectivity of scientific representations as well. In this article I argue that the two main naturalistic alternatives are mistaken, thus pointing to the conclusion that no substantive naturalistic theory of scientific representation will succeed. I am certainly not the first to criticize similarity and isomorphism theories. For instance, Cummins (1989, Chap. 3) discusses and rejects similarity theories of mental representation, and Downes (1992) criticizes isomorphism as characterizing the empirical adequacy of scientific theories. I focus my critique on similarity and isomorphism as theories of *scientific representation*, and I argue that both fail for precisely the same set of reasons.

2. Representation naturalized: similarity and isomorphism

What sort of factual relation must hold between A and B for A to represent B? For instance, what relation must hold between the graph of a bridge, and the bridge it represents? It is obvious that not any arbitrary relation between A and B will do: for there are all sorts of relations that obtain between A (e.g. the graph) and B (e.g. the bridge), which are irrelevant to the representational relation itself—such as “being an artefact”, or “being at least 10 cm long”. The success of the project of naturalizing representation is crucially dependent upon finding a suitable type of relation that can fill in this role. For the theory of representation to be substantive in my sense it is required that this relation obtains universally between the source and the target, in all instances of successful scientific representation.

Two accounts have been available in the literature for some time: similarity and isomorphism. Ronald Giere (1988, 1999a) has defended the importance of similarity for representation, which has also been stressed for instance by Aronson *et al.* (1993). Bas van Fraassen (1992, 1994) has concentrated on the virtues of isomorphism; and other writers in the structuralist tradition, including most prominently Brent Mundy (1986), have appealed to weakened versions of isomorphism.

We may enunciate the corresponding theories as follows:³

The *similarity conception of representation* [sim]: A represents B if and only if A is similar to B.

The *isomorphism conception of representation* [iso]: A represents B if and only if the structure exemplified by A is isomorphic to the structure exemplified by B.

Similarity is a generalization of resemblance. Two objects resemble each other if there is a significant similarity between their visual appearance. [sim] does not assert that resemblance is a necessary and sufficient condition for representation; it is a weaker condition, which neither requires nor includes similarities in visual appearance, or a threshold “significant” amount of similarity. The following is typically assumed: A and B are similar if and only if they share a subset of their properties. In accordance with this *identity-based theory* similarity is reflexive (A is maximally similar to itself), and symmetric (if A is similar to B, on account of sharing properties p_1, p_2, \dots, p_n , then B is similar to A on the same grounds); but non-transitive (A may share p_1 with B, and B may share p_2 with C, without A and C sharing any property—other than the property of sharing a property with B!).

Isomorphism is well defined only as a mathematical relation between extensional

structures. Hence the above definition presupposes that any two objects that stand in a representational relation exemplify isomorphic structures. The notion of structure-exemplification turns out to be ridden with difficulties; but the definition has the virtue that it makes sense of object-to-object representation outside pure mathematics. The claim that two physical objects A and B are isomorphic is then shorthand for the claim that the extensional structures that A and B exemplify are isomorphic. In what follows “A” will indistinguishably denote the source and the structure that it exemplifies, and “B” will denote the target and the structure that it exemplifies. Isomorphism then demands that there be a one-to-one function that maps all the elements in the domain of one structure onto the elements in the other structure’s domain and vice versa, while preserving the relations defined in each structure. Hence A and B must possess the same cardinality. More precisely, suppose that $A = \langle D, P_j^n \rangle$, $B = \langle E, T_j^n \rangle$; where D, E are the domains of objects in each structure and P_j^n and T_j^n are the n -place relations defined in the structure. A and B are isomorphic if and only if there is a one-to-one and onto mapping $f: D \rightarrow E$, such that for any n -tuple $(x_1, \dots, x_n) \in D$: $P_j^n[x_1 \dots, x_n]$ only if $T_j^n[f(x_1), \dots, f(x_n)]$; and for any n -tuple $(y_1, \dots, y_n) \in E$: $T_j^n[y_1, \dots, y_n]$ only if $P_j^n f^{-1}[y_1, \dots, f^{-1}(y_n)]$. In other words, an isomorphism is a relation preserving mapping between the domains of two extensional structures, and its existence proves that the *relational framework* of the structures is the same.⁴

It is possible in general to understand isomorphism as a form of similarity. For suppose that A and B are isomorphic; then they share at least one property in common, namely, their *relational framework*. Hence two isomorphic structures are similar, because their relational frameworks are identical. So two objects that exemplify isomorphic structures are *ipso facto* structurally similar. The similarity in case (2) between the bridge and its graph is precisely of this type. This is *prima facie* an interesting advantage that similarity enjoys over isomorphism. For neither similarity nor resemblance can in general be reduced to isomorphism. Judgements of similarity unproblematically apply to any sort of objects, including for instance perceptual experiences, and it is unclear to say the least how these experiences could be said to exemplify structures at all. Whether or not such reduction is ultimately theoretically possible, in no ordinary context are we able to translate judgements of similarities in, say, taste, to isomorphisms between anything like “taste structures” of different types of food. Analogously for most judgements of resemblance. The basic problem is that similarity and resemblance are ordinarily and unproblematically applied to both response-dependent and intensionally defined properties, while isomorphism is not.

But what about those cases of representation where the source and target can be ascribed an explicit structural exemplification? Arguably, many scientific representations are of this sort. But even in these cases the reduction of similarity to isomorphism is typically only possible conditional on the appropriate exemplification of structure. Two objects may be similar in sharing just some of their properties, such as, i.e. the colour distribution of their surfaces. So only the structures defined by the colour relation may be isomorphic. While it is correct to claim that such objects are similar, the isomorphism claim must be restricted to the specific properties shared.

Let us then suppose that either [iso] or [sim] were correct. It follows that to establish in cases 1–4 that the source is a genuine representation of the target, we need to investigate the properties of the source and those of the target, and the relationship between them. No further investigation is required. Representation will obtain if the right type of relational facts obtain between A and B, independently of any agent’s judgements on the matter. Thus if we can show [iso] or [sim] to be correct we will *ipso*

facto have naturalized the notion of scientific representation. This, I think, is to a large extent the motivation and driving force behind the [iso] and [sim] conceptions.

For example, the aim to naturalize scientific representation is clear in Giere's work and may in fact be taken as a constant in his intellectual trajectory (see, e.g. Giere, 1988, 1999a, b, forthcoming). But we must be very careful to distinguish clearly the different strands of naturalism present in his work. There are at least two clearly distinct claims: there is a weak form of naturalism that merely claims that science can study representation; and a stronger form of naturalism, which I employ in this article, that claims that the relation of representation does not involve in any essential way agent's intentions and value judgements, but appeals only to the facts. Over the years Giere has moved from a defence of both claims to a defence of the weaker claim only. So his recent defence of naturalism (Giere, 1999a, forthcoming) is compatible with my rejection of the naturalistic theories of representation discussed in this article. (I am indebted to illuminating discussions with Ron Giere and Bas van Fraassen on this point.)

3. Means and constituents of representation

I want to first distinguish the *means* and the *constituents* of representation. In practice the main purpose of representation is surrogative reasoning (Swoyer, 1991). Suppose, for instance, that an object A represents an object B; then A must hold some particular relationship to B that allows us to infer some features of B by investigating A. Take for instance the example of the phase space representation of the motion of a classical particle. The graph may be similar in respects a, b, c to the particle's motion; and when we reason about the graph in order to infer features of the particle's motion we do so by studying precisely that similarity. The means of the representation are thus those relations between A and B that we actively make use of in the process of inquiring about B by reasoning about A. Notice crucially that an object A or system may hold more than one type of relation to another B, but at any one time only one of these will be the means of representation. For example, a phase space graph of the motion of a paper ball in air may be both structurally isomorphic to the ball's motion in space, and in addition similar to the ball in being drawn on the same type of paper. The similarity obtains but is not the means of the representation in this case (although there are circumstances in which it could be, for example, if we were investigating the properties of paper, not motion!).

Thus there may be a great variety of means by which representation does its work: isomorphism and similarity are just two common ones, but there are others, such as exemplification, instantiation, convention, truth. In addition, the means of representation are not exactly transparent: no source wears its means of representation "on its sleeve". In many cases the actual means of a representation may be opaque to the uninitiated. Consider a bubble chamber photograph, an astronomical chart, or an equation of motion. To correctly understand what and how these sources ground inferences about their representational targets invariably requires informed and skilful judgement. Normally only one among the many relations obtaining between A and B is intended to provide grounds for such inferences. So much is common lore, particularly in the philosophy of art. It is surprising therefore that the implications of this simple observation regarding the nature of scientific representation seem not to have been picked out. In particular, I will argue, it follows that neither [iso] nor [sim], on their own, can account for the *means* of scientific representation.

At this point the distinction between the means of representation and its con-

stituents may be drawn as follows. The fact that we use a particular relation (say, similarity) between A and B to, say, infer B's properties by reasoning about A's properties, should not be taken to mean that this relation is what *constitutes* the representation by A of B. There could be a deeper, hidden relation between A and B. Suppose that A (for instance, a phase space structure) represents B (the motion of a particle in space) in virtue of an isomorphism. This appears to be consistent with the fact that sometimes in reasoning successfully about B on the basis of A we need not employ or refer explicitly to the isomorphism of A and B, but are able to use some other relationship instead. For instance, on a particular occasion it may be possible to investigate the properties of a particle's motion merely by investigating its similarity (i.e. shared properties, such as for instance the appearance of randomness) with its phase space graph. It would appear then that in this case the means of the representation (similarity) fail to agree with its deeper constituents (isomorphism).

We may then consider the following definitions:

Means of representation: At any time, the relation R between A and B is the means of the representation of B by A if and only if, at that time, R is actively considered in an inquiry into the properties of B by reasoning about A.

Constituents of representation: The relation R between A and B is the constituents of the representation of B by A if and only if R's obtaining is necessary and sufficient for A to represent B.

The distinction opens up a promising avenue for defending [iso] and [sim]. One could take [sim] ([iso]) as the basic constitutive notion at the heart of representation, which warrants the representational relation, while accepting that isomorphism (similarity) may be employed as useful means once [sim] ([iso]) has been established. For example, the isomorphism between a phase space structure and the motion of a particle could be said to be the efficient means for applying the relevant similarities of structure that warrant [sim], and hence representation. Or, alternatively, the observed similarity between two bridges may be taken to be merely an efficient means for us to take cognitive advantage of the deep structural isomorphism existing between the bridge's structures: the similarity is only a means to more efficient reasoning, but it is the isomorphism that is actually warranting the representational relation, in accordance with [iso]. Hence [sim] ([iso]) may fully characterize the constituents of the relation of representation, while failing to characterize its means.

4. Five arguments against similarity and isomorphism

I will now present five arguments against [sim] and [iso]. Some of these arguments were first advanced by Suárez (1999) and have been reiterated by Frigg (2002). The first argument is the simple empirical observation that neither [sim] nor [iso] can be applied to the full variety of uses of representational devices that crop up in the practice of science. Hence an analysis of the means of representation in terms of just one of these conditions would be unduly restrictive and local. However, as I pointed out above, the defenders of [iso] and [sim] have an easy retreat: they can argue that [iso] and [sim] are meant as substantive theories of the *constituents*, not the means of the representational relation; they are meant to describe the relation that must obtain between A and B for A to represent B, independently of what relations are actually employed by enquirers in drawing inferences about B on the basis of A. The retreat is perfectly honourable and

legitimate, for it is in line with the pretensions of a substantive naturalized theory of representation.

However, four other arguments show that even in those cases where [iso] and [sim] apply, the analysis they yield is incorrect; in other words, the isomorphism and similarity conceptions cannot on their own *constitute* representation. The second argument is that [iso] and [sim] lack some of the logical properties of representation. The third argument is that they do not allow for misrepresentation or inaccurate representation. The fourth argument is that [sim] and [iso] are not necessary for representation—they fail in some cases of successful representation. The fifth and final argument is that neither [iso] nor [sim] can be sufficient for representation, because they leave out the essentially directionality of representation.

4.1. *The argument from variety: [sim], [iso] do not apply to all representational devices*

Although similarity and isomorphism are among the most common means of representation in science neither one, on its own, covers even nearly the whole range. We have some firmed up intuitions, I think, about the means of representation in the four cases mentioned.

Case 1 (Toy bridge representation): Similarity is almost always the means for concrete physical representations of concrete physical objects. An engineer's toy bridge may be similar to the bridge that it represents in the proportions and weights of the different parts, the relative strengths of the materials and the geometric shape. It is by reasoning on the basis of these similarities that the source does its representational work. There are also important dissimilarities, such as size, which make the representation only a partially successful one, but similarity again seems to be a good guide to determining which parts are representational and which aren't. By contrast isomorphism, which is well defined only as a relation between mathematical structures, does not apply directly to the relation between two physical objects described in case 1. But it does apply to some abstract structures that are exemplified by these two objects, such as their geometric shape.

However, the representational use of the toy bridge is almost always grounded on actual reasoning about its properties, along with those of the real bridge, and not on the properties of the structures exemplified by either bridge. The means of the relation of representation are not in this case captured by the [iso] conception because this conception misidentifies its relata, which are the physical objects themselves, and not the structures exemplified. To make this point vivid suppose, for instance, that two concrete toy bridges exemplify exactly the same geometric structure, isomorphic to that of the larger bridge. We typically treat these two bridges as two different means and as distinct representations of the same object, but an isomorphism analysis of the means of representation does not allow us to do that: the relationship R that each toy bridge holds to the larger bridge is exactly the same.⁵

Case 2 (Graph of bridge): The range and depth of the dissimilarities between the source and the target become greater in this case: a piece of paper containing the graph of a bridge is only similar to the bridge it represents with respect to the geometric shape and proportions between the different points; nothing else is interestingly similar. This "similarity of structure" is better captured by the alternative conception [iso]. Maps, plans and graphs are typical cases where isomorphism is the means of scientific representation.

Case 3 (Billiard-ball model): This case appears to be harder for both conceptions.

A system of billiard balls is not *prima facie* in any relevant sense similar to any state of nature. We may anyway refer to the relation of similarity or isomorphism that can obtain between two token instances of these things. If so we must make sure that we are referring to a similarity between the *dynamical* properties of the systems, collectively taken, as a system of billiard balls is similar to a system of gas molecules only in its dynamical properties, and in no properties of the entities taken individually at any one time—other than their elasticity. *Mutatis mutandis* for isomorphism: this obtains only between the mathematical structures exemplified by the dynamics of the systems, and not between the structures exemplified by the individual entities.

Case 4 (Quantum state diffusion equation): This case is simply not covered by the similarity analysis at all. A mathematical equation, written down on a piece of paper, represents a certain physical phenomenon but is not similar to it in any relevant respect. If the equation is dynamical, one may focus on the phase space structure defined by the equation, and on that structure which is best exemplified by the phenomenon: if the equation is an accurate representation of the phenomenon, isomorphism will obtain between them and, as noted in Section 1, isomorphism is a case of similarity.

But even [iso] is problematic here. In most cases of mathematical representation it seems far-fetched to assume that the means of the representation is an isomorphism. It trivially is the case that the dynamic phase space structure exemplified by a differential equation must be isomorphic to the dynamic structure exemplified by the phenomenon, if the equation accurately represents the phenomenon. But when scientists reason about a differential equation in order to inquire into the phenomenon it represents, they rarely include an investigation of the formal properties of these structures. What they actively do is look for solutions to the equation given certain boundary conditions, and then check whether some parameters of those solutions correspond to observed features of the phenomena. The isomorphism which obtains is not what they explicitly reason about, so it is not in this case the means of the representation.

4.2. *The logical argument: [sim], [iso] do not possess the logical properties of representation*

A substantive theory must make clear that scientific representation is indeed a type of representation; i.e. that it shares the properties of ordinary representation. Representation in general is an essentially non-symmetric phenomenon: a source is not represented by a target merely in virtue of the fact that the source represents the target. There may be contexts in which symmetry obtains, but even there it is not automatic: merely because a photograph portrays a person, in some context, it does not follow that the person stands for the photograph, in the same context; merely because an equation represents a phenomenon, the phenomenon cannot be said to stand for the equation. Representation is also non-transitive and non-reflexive (as a matter of fact representation is probably in addition irreflexive, asymmetric and intransitive; but I will not here need to argue that much). A theory of scientific representation must do justice to these features.

Nelson Goodman (1976, pp. 3–10) used these logical properties of representation to argue against resemblance theories, and his argument carries over against [sim] and [iso]. I shall pursue here an illuminating analogy with painting—a particularly apt analogy in this context, as [iso] and [sim] both assume that scientific representation is essentially an object-to-object relation rather than word-to-object, or mental state-to-object relation. That is, both [iso] and [sim] assume that both relata of the relation of representation are similarly structured entities endowed with properties. This explains

why [iso] and [sim] have been particularly attractive to defenders of the semantic view of theories, since on this view theories are not linguistic entities but structures.⁶

The argument is, however, independent of the analogy, and is in no way exhausted by it. The purpose of the analogy is to call attention to the logical properties of object-to-object representation in general, thus suggesting that scientific representation must display these properties too. It could however turn out that scientific representation is not a kind of object-to-object representation, or not entirely so; but this is a possibility that would *ipso facto* refute the [iso] and [sim] conceptions that I criticize here. To defeat the argument, one would have to show that [iso] and [sim] have the logical properties of representation in general, which I think is patently not the case.

Representation is non-reflexive: Diego Velázquez's portrait of Pope Innocent X represents the Pope as he was posing for Velázquez but it does not need to represent the portrait itself. Admittedly, Velázquez astounded the world with the striking built-in reflexivity of *Las Meninas*, which represents among other things, the act of its being painted. A creative obsession with representing the very elusive act of representation is part of art since at least the Quattrocento. But even in these cases the representation typically adds to the object, and also subtracts from it: the source and the target are not exactly identical.

It would be equally wrong to claim that the Pope represents the painting. We may put aside issues about whether a non-existing object can be said to represent: even when the Pope was sitting down posing for Velázquez it would not have been right to claim that he represented the painting. Representation is non-symmetric, at least, since it is often one-way. Apparent cases of symmetry, such as some of Escher's drawings, or two mirrors placed opposite and reflecting each other, turn out to be cases where there is a distinct representational relationship going each way.

The recently deceased painter Francis Bacon was obsessed with Velázquez's portrait of Innocent X, and produced a large number of variations of his own, all of them intending to represent the Velázquez canvass. The Bacon portraits represent not the Pope, but the Velázquez canvass. Or, suppose that a tourist takes a photograph of the room where *Las Meninas* hangs in the Prado; that photograph represents the canvas, not the Spanish Royal Family. Representation is at least non-transitive, since apparent cases of transitivity turn out on inspection to involve different representational relations between A and B and between B and C.

However, similarity is reflexive and symmetric, and isomorphism is reflexive, symmetric and transitive. A glass of water is similar to itself, and similar to any other glass that is similar to it. *Mutatis mutandis* for isomorphism: a geometric structure (a square) is isomorphic to itself, and always isomorphic to any other structure (another square of perhaps a different size) that is isomorphic to it, and isomorphic to any structure that is isomorphic to a structure that is isomorphic to it (an even larger square).

4.3. *The argument from misrepresentation: [sim], [iso] do not make room for the ubiquitous phenomena of mistargeting and/or inaccuracy*

Misrepresentation is an ubiquitous phenomenon in ordinary-life representation. It comes in two varieties. There is first the phenomenon of mistaking the target of a representation, or as I call it *mistargeting*: often we mistakenly suppose the target of a representation to be something that it actually does not represent. Suppose that a friend of mine has disguised himself to look roughly similar to Pope Innocent X in the relevant

respects. In seeing the Velázquez canvass for the first time I am struck by this resemblance and, ignoring the history and true target of the representation, I go on to suppose that the Velázquez represents my friend. This is a clear case of misrepresentation, but there is no failure of similarity to explain it. Indeed misrepresentation by accidental similarity would be impossible if [sim] were true, precisely because similarity would then warrant representation. Exactly the same argument goes *mutatis mutandis* for isomorphism, and it is an argument that can be easily transferred to cases of scientific mistargeting. Consider the case of the quantum state diffusion equation:

$$d|\psi\rangle = -i/h H|\psi\rangle dt + \sum_j \langle L_j^* \rangle L_j - \frac{1}{2} \langle L_j^* \rangle \langle L_j \rangle |\psi\rangle dt + \sum_j (L_j - \langle L_j \rangle) |\psi\rangle dt.$$

This equation represents the evolution of the quantum vector state of a particle subject to a diffusion process. (The first term is just the usual Hamiltonian in the linear Schrödinger equation, the other two terms account for random diffusion and interaction with a larger environment.) A mathematician who knew nothing about quantum mechanics would be able to solve this equation for some boundary conditions; by accident the motion described may correspond to a particular classical particle's Brownian motion. This accidental fact on its own does not turn the equation into a representation of the particle's motion, however, because the essential directionality of representation is missing.

The point has been argued persuasively in the general case by Putnam (1981, Chap. 1) and in the scientific case by van Fraassen (1994, p. 170), and need not be rehearsed in detail here. It has long been noted within the philosophy of art too. Thus Richard Wollheim writes:

The connection between seeing-in and representation was noted by theorists of representation both in antiquity and in the Renaissance. Yet almost to a person these thinkers got the connection the wrong way round: they treated seeing-in as—logically and historically—posterior to representation. For they held that, whenever we see, say, a horse in a cloud, or in a stained wall, or in a shadow, this is because there is a representation of a horse already there—a representation made, of course, by no human hand. These representations, which would be the work of the gods or the result of chance, wait for persons of exceptional sensitivity to discern them, and then they deliver themselves up. (Wollheim, 1987, p. 54)

For Wollheim, like Putnam, van Fraassen and myself, the skill and activity required to bring about the experience of seeing-in (the appreciation by an agent of the “representational” quality of a source), is not a consequence of the relation of representation but a condition for it.

Similarly for scientific representation. For suppose Putnam, van Fraassen and Wollheim were all wrong on this point: a mathematician's discovery of a certain new mathematical structure (defined by a new equation perhaps) isomorphic to a particular phenomenon would amount to the discovery of a representation of the phenomenon— independently of whether the mathematical structure is ever actually applied by anyone to the phenomenon. The history textbooks would have to be rewritten so that it was Riemann, not Einstein, who should get credit for first providing a mathematical representation of space-time.

The second form of misrepresentation is the even more ubiquitous, perhaps universal, phenomenon of inaccuracy. Most representations are to some degree inaccur-

ate in some or other respects. [iso] cannot account for inaccurate representation at all. For on this conception either a model is a representation of, and thus isomorphic to, its target, or it is not a representation at all. [sim] requires that the target and the source must share some although not necessarily all their properties. Hence [sim] can account for the type of inaccuracy that arises in an incomplete or idealized representation of a phenomenon, i.e. one that leaves out particularly salient features such as the highly idealized representation of classical motion on a frictionless plane. But this will not always help to understand inaccurate representation in science, where the inaccuracy is much more often quantitative than qualitative. For example, Newtonian mechanics, without general relativistic corrections, can at best provide an approximately correct representation of the solar system. Some motions would not be quite as predicted by the theory, even if all features of the solar system were to be accounted for. The interesting question is not what properties fail to obtain, but rather how far is the divergence between the predictions and the observations regarding the values of the properties that do obtain. [sim] offers no guide on this issue.

4.4. *The non-necessity argument: [sim], [iso] are not necessary for representation—the relation of representation may obtain even if [sim], [iso] fail*

It is trivial that any object is in principle similar to any other object. In fact the point is often made that if all logically possible properties are permitted, then any object is similar to any other object in an infinite number of ways, i.e. there is an infinite number of properties that we can concoct that will be shared between the objects (“being on this side of the moon”, “being neither black nor blue”, etc.). If so, similarity would be necessary for representation but in a completely trivial way. For it would not only be a necessary condition on representation but also on non-representation.

The defender of similarity may retort that it is not fair to include those logically possible shared properties that have nothing whatever to do with the representation itself (such as “being on this side of the moon”). A restriction is needed here to only those properties or aspects of the source and the target that are “relevant” to the representational relation: A represents B if and only if A and B are similar *in the relevant respects*. It is not the case that any source is in principle trivially similar in the relevant aspects to what it represents. Suppose that I am interested in representing in a painting the colour of the ocean in front of me. I may represent the ocean by painting some blue and green stripes on a piece of paper. Representation obtains in this case if the colours on my paper are similar to those of the ocean, and it fails otherwise: that is the only relevant property. Any other logically possible similarity, such as “being on this side of the moon”, is irrelevant to this particular representation.

There are two important objections to this move. First, what is the criterion of relevance invoked here? This criterion must presumably link relevance to the representational relation itself, for otherwise there would be no reason to expect *relevant similarity* to be necessary for representation. The shared properties that are relevant are precisely those that pertain to the representation. So, we obtain that, A represents B if and only if A and B are similar in those respects in which A represents B. However illuminating this may be about the actual use of similarity, it is circular as an analysis of representation!

The defender of similarity might retort that ‘relevance’ is a fully intuitive notion of straightforward application in practice; a primitive notion in no need of further analysis. That this is not so is made most vivid in the analogy with art, and to illustrate this point

I like to invoke *Guernica*, the well-known painting by Picasso. There are similarities between parts of this painting and many objects, such as a bull, a crying mother holding up a baby, an enormous eye. They all seem undeniably relevant to the representational content of the painting, if any similarities are, yet none of these similarities is a good guide to the actual targets of the representation. There are at least two targets. Picasso was interested in representing the first ever carpet-bombing of an entire civilian population: the bombing, under Franco's consent if not direct orders (Preston, 1993, Chap. 9), of the Basque town of Guernica by Hitler's Condor Legion and Mussolini's Aviazione Nazionale in 1937. In addition, *Guernica* represents the threat of rising Fascism in Europe, which is the reason why it was hugely effective in bringing world attention to the Spanish Republic's cause. This is all historically well documented.⁷ The point is that none of the targets of *Guernica* can be easily placed in the relevant similarity relation with the painting, and *mutatis mutandis* for isomorphism.⁸

The case in science is not significantly different. An equation—i.e. the actual physical signs on the paper—is as dissimilar as it could be from the phenomenon that it represents. *Mutatis mutandis* for isomorphism, as we have already seen in the case of inaccurate representation. We are perfectly happy with the claim that Newtonian mechanics provides a representation of the solar system, even if it is clear that Newtonian mechanics, without general relativistic corrections, is empirically inadequate and non-isomorphic to the phenomena of planetary motion. A possible retort on behalf of [iso] and [sim] is that we should concentrate entirely upon the subset of properties, or the substructure, that corresponds to those motions that are correctly predicted. But in cases of quantitative inaccuracy this normally will not help. Newtonian mechanics arguably does not describe *any* actual planetary motion in a quantitatively accurate way.

4.5. *The non-sufficiency argument: [sim], [iso] are not sufficient for representation—the relation of representation may fail to obtain even if [sim], [iso] hold*

The previous four arguments already point to a feature of representation that is not captured by the [iso] or [sim] analyses: the essential directionality of representation. This was perhaps most apparent in the argument from misrepresentation: the object that constitutes the source of a model has no directionality per se, but in a genuine representational relation the source *leads* to the target. Neither similarity nor isomorphism can capture this capacity of the representational relation to take an informed and competent inquirer from consideration of the source to the target. But it is this feature that lies at the heart of the phenomenological non-symmetry of representation. Consider for instance two identical glasses. They share all their (monadic) properties, and are hence as similar as they could be. But neither of them leads to the other unless they are in a representational relation, and then only that which is the source will have the capacity to lead to the target. Or consider the trajectory in phase space described by the state vector of a quantum particle. Unbeknownst to us this trajectory may well be isomorphic to the motion in physical space of a real classical particle. But unless the phase space model is intended for the particle's motion, the representational relation will fail to obtain. Hence neither similarity nor isomorphism are sufficient for representation.

There is an additional reason why isomorphism is not sufficient for scientific representation. It is related to the notion of structure-exemplification. Goodman (1976, pp. 52ff.) provided an analysis of the notion of exemplification as a special class of representation: if *x* exemplifies *y* then *x* denotes *y* and *y* denotes *x*; but *x* may denote *y* without exemplifying it—exemplification requires denotation *both* ways. My sweater

exemplifies red if and only if it both denotes red and is denoted by red (i.e. the sweater is used to refer to red and also *is* red).

Now let us suppose that this analysis of exemplification goes through for structural representation. Then whenever object x exemplifies structure y it both represents y and is represented by y . It follows then that for an object A to represent some object B by means of [iso], the structure exemplified by A must be isomorphic to the structure exemplified by B. But that just means, if the supposition is right, that for A to represent B there must be a structure that represents A isomorphic to a structure that represents B. And we will now want to ask how the structures represent the objects in the first place.

For instance, the quantum state diffusion equation for a localizing particle describes a random walk motion in a phase space structure. This structure represents not the particle's motion, but a representation of it, namely, the motion of the vector in Hilbert space that corresponds to the state of the particle. But representing a representation of x is in no way equivalent to representing x ; and we are left with the question of how x is mathematically represented in the first place. So isomorphism is not in general sufficient for representation. Perhaps paradoxically the case of representation of a well-established physical phenomenon by means of a differential mathematical equation is the hardest case for [iso] to accommodate. (Note that I am not claiming that isomorphism is irrelevant to mathematical modes of representation in science, only that it does not constitute representation; it may for instance be very useful in establishing the *accuracy* of a representation.)

5. The amended versions fare no better

A recurrent theme in these arguments, which became explicit in the discussion of the non-sufficiency argument, is the appeal to the essential directionality of representation: a necessary condition for A to represent B is that consideration of A leads an informed and competent inquirer to consider B. I will refer to A's capacity to lead a competent and informed enquirer to consider B as the *representational force* of A. Representational forces are relational properties of sources in particular contexts of inquiry. They are determined at least in part by correct intended uses, which in turn are typically conditioned and maintained by socially enforced conventions and practices: A can have no representational force unless it stands in a representing relation to B; and it cannot stand in such a relation unless it is intended as a representation of B by some suitably competent and informed inquirer.

Note that I am careful to refer to the essential *directionality*, not *intentionality*, of representation. There has been an important debate in the philosophy of mind in recent decades that assumes that a source's representational force is nothing but the intentionality of the mental state of an agent that employs the source to represent some target. This assumption is friendly to my critical analysis, since intentionality has the right properties to ground the arguments that I propose in this article against similarity and isomorphism. There is no doubt that the connection between representational force and intentional states requires further analysis; however, I neither need nor wish to make this assumption here. At best, given the present lack of consensus about what intentionality may be, this assumption would be akin to trying to explain a child's ability to ride a bicycle by appealing to his sense of balance: even if nobody doubts that there is a connection, it is not very explanatory since we do not have a clear understanding of how children develop their sense of balance. Analogously, it cannot be sound methodology

to invoke a difficult and obscure notion (intentionality) in order to explain a difficult but not particularly obscure human activity (representation). That is why, at this stage at least, I prefer a plain intended-use theory of representational force that leaves room for further specification, which may or not include intentionality.⁹

Can [iso] and [sim] be made to work by simply amending them to account for this directional component? The amended versions would look as follows:

[sim]': A represents B if and only if (i) A is similar to B and (ii) the representational force of A points to B.

[iso]': A represents B if and only if (i) the structure exemplified by A is isomorphic to the structure exemplified by B and (ii) the representational force of A points to B.

The first thing to notice about these amended versions is that they abandon the aim of naturalizing representation. Representation can no longer be established by means of a scientific investigation of the facts of the matter—for there are elements in the relation of representation, namely, the *representational forces* in part (ii), that essentially involve value judgements, and are not reducible to facts.¹⁰

But in fact, [sim]' and [iso]' cannot be correct. Certainly the additional clause stipulating the correct intended use of the representation turns conditions [iso]' and [sim]' into sufficient conditions for representation, and the *non-sufficiency* argument no longer applies. Depending on how we explicate intended use, the logical argument might also lose its force. But the other arguments still apply. The *non-necessity* argument is, if anything, strengthened as the necessary conditions on representation are now stronger. The *argument from variety* shows that neither [iso]' nor [sim]' can describe all the means of representation; while the *misrepresentation* and *non-necessity* arguments show that they do not provide a substantial theory of the constituents of representation. Simply adding further conditions to [iso] or [sim] to make room for the essential directionality of representation will not help.

6. Weakening similarity and isomorphism

The prospects for a substantial naturalistic theory of representation seem bleak. Certainly [sim] and [iso] are non-starters. In this final section I take a look at a number of attempts to weaken the conditions on representation imposed by [iso] and [sim]. These programmes are either being tentatively developed at present or could be developed. So my conclusions have a correspondingly tentative and provisional character.

6.1. Similarity without identity

The problem then lies not with what [iso] and [sim] lack but with what they have. We must try to subtract from, not add to, these conditions. One assumption that was built into [sim] is the identity-based theory of similarity. This theory seems natural, gives a high level of precision to the concept, and makes it possible for us to quantify and measure degrees of similarity between objects (as ratios of properties shared). But it may be mistaken.

Evidence has been reported (by Eileen Way, for example, in a talk at the Las Cruces modelling conference, New Mexico, January 2002) within experimental work in cognitive psychology for a non-identity-based understanding of similarity, which empha-

sizes the essential role of contextual factors and agent-driven purposes in similarity judgements. (In the terminology of this article, this turns [sim] into a non-naturalistic theory.) Let us suppose that similarity between two objects is not simply a case of sharing a property, but a more complex contextual relation. We don't have a very good understanding of what this relation may be, but Eileen Way argues that on such theories of similarity there is typically no reason to expect similarity judgements to be symmetric: the fact that A is similar to B does not *ipso facto* require B to be similar to A. If Way is right the *logical* argument does not cut as strongly against similarity as it seemed. But it applies nonetheless. For however similarity is conceived, it must be reflexive. If something is not similar to itself then it is not similar to anything else. Any theory of similarity must concede this: similarity comprises identity; identity is a limiting case of similarity. Here representation and similarity definitely depart, for the vast majority of representations patently do not represent themselves.

However, the combination of Giere's emphasis on the essentially pragmatic character of similarity judgements (Giere, 1988) with Way's non-identity-based understanding of similarity would undeniably bring similarity and representation closer. And indeed Giere has recently proposed to understand representation as a four-place *activity*. As he writes: "The activity of representing, if thought as a representation at all, should have at least four places with roughly the following form: 'S uses M to represent W for purposes P'" (Giere, forthcoming). This theory would be successful to the extent that it builds the source's representational force, with all its normative import, into the relation of representation itself. That would turn it into a non-naturalistic theory in the terminology of this article. The non-sufficiency argument would have no force against such a theory, and neither would the mistargeting part of the argument from misrepresentation, or the non-symmetry part of the logical argument. Reflexivity and the non-necessity argument would remain the standing blocks for this interesting non-naturalistic theory.

6.2. Homomorphism

Elizabeth Lloyd, a prominent defender of the semantic view, suggests that "in practice the relationship between theoretical and empirical model is typically weaker than isomorphism, usually a homomorphism, or sometimes even a weaker type of morphism" (Lloyd, 1988, Chap. 2, n. 2). Although Lloyd is not in this passage specifically referring to representation, the [iso] condition does get weakened in a variety of ways, which solve some but not all of the problems that I have raised. For instance, following the pioneering work of Krantz *et al.* (1971), Brent Mundy employs the notion of homomorphism, and shows how to apply it to measurement theory, space-time geometry and classical kinematics. We say that an extensional structure A is faithfully homomorphic to an extensional structure B if and only if there is a function that maps all the elements in the domain of A into the elements in B's domain, while preserving the relations defined in A's structure. More precisely, suppose that A and B uniquely exemplify the structures $\langle D, P_j^n \rangle$ and $\langle E, T_j^n \rangle$; where D, E are the domains of objects in each structure and P_j^n and T_j^n are the n -place relations defined in the structure. Then A is *faithfully homomorphic* to B (Mundy, 1986, p. 395) iff there is a mapping $f: D \rightarrow E$, such that for any n -tuple $(x_1 \dots, x_n) \in D: P_j^n[x_1, \dots, x_n]$ if and only if $T_j^n[f(x_1), \dots, f(x_n)]$. The correspondingly weakened version of [iso] is:

The *homomorphism conception of representation* [homo]: A represents B if and only if the structure exemplified by B is homomorphic to the structure exemplified by A.

An homomorphism is, unlike an isomorphism, neither one-to-one nor onto, so the cardinality of A and B may differ. This feature was notoriously used by Krantz *et al.* (1971) to show that [homo] rather than [iso] is appropriate for theories of measurement. The important advantage that [homo] enjoys over [iso] is then the ability to deal with partially accurate models. Parts of a source may not represent any of the aspects of the homomorphic target. So the hope is that [homo] will be able to refute the part of the argument from misrepresentation that refers to inaccurate representation, and its consequences for the non-necessity argument. The solar system may only be represented by the part of the Newtonian model that asserts the number of planets and their average proximity to the sun, without specifying their precise motions. The highly developed structural theory of measurement as homomorphism into the real number continuum allows [homo] to provide precise estimates for these numbers (Krantz *et al.*, 1971; see also Díez, 1997a, b). It seems clear that the move to [homo] weakens the non-necessity argument (although interestingly it does not dispel the force of the art analogy in that argument).

However, all the other arguments apply against [homo] too. This includes the argument from variety; the mistargeting part of the argument from misrepresentation; and the non-sufficiency argument. The logical argument is significantly weakened but not avoided: homomorphism is neither symmetric nor transitive, but it is reflexive.

6.3. *Partial isomorphism*

Another proposal to weaken [iso] may be provided by Mikenberg *et al.*'s (1986) notion of partial structure, and the corresponding notion of partial isomorphism introduced by Bueno (1997). A partial structure $\langle D, R_{i1}, R_{i2}, R_{i3} \rangle$ defines for each relation R_i a set of n -tuples that satisfy R_i , a set of n -tuples that do not satisfy R_i , and a set of n -tuples for which it is not defined whether they satisfy R_i or not. Given two partial structures $A = \langle D, R_{i1}, R_{i2}, R_{i3} \rangle$ and $B = \langle E, R'_{i1}, R'_{i2}, R'_{i3} \rangle$ “the function $f: D \rightarrow E$ is a partial isomorphism if (i) f is bijective, and (ii) for every x and $y \in D$, $R_{i1}(x, y)$ if and only if $R'_{i1}(f(x), f(y))$ and $R_{i2}(x, y)$ if and only if $R'_{i2}(f(x), f(y))$ ” (Bueno, 1997, p. 596; French & Ladyman, 1999, p. 108). The corresponding theory of representation would then be:

The *partial isomorphism conception of representation* [partial iso]: A represents B if and only if the structure exemplified by A is partially isomorphic to the structure exemplified by B .

The advocates of partial isomorphism argue that the introduction of R_{i3} serves to accommodate the partiality and openness of the activity of model building. That may be so, but as a theory of representation [partial iso] fares even worse than [homo]. Since according to (i) f is bijective, it follows from (ii) that $R_{i3}(x, y)$ if and only if $R'_{i3}(f(x), f(y))$, and hence partial isomorphism reduces to three separate isomorphisms. So it remains to be seen whether [partial iso] can avoid the inaccuracy part of the argument from misrepresentation, and correspondingly weaken the non-necessity argument. Even if this could be done, [partial iso] would be at a disadvantage with respect to [homo] since the logical argument weights even more strongly against [partial iso]: partial isomorphism, unlike homomorphism, is symmetric.

6.4. *Structural representation without isomorphism*

Other writers within the structuralist tradition have been more cautious. It does not follow from the claim that theories (or models) are, or contain, structures that the

relation that constitutes representation is a structural one. The arguments that I have presented in this article suggest that we should look elsewhere for the constituents of representation, perhaps even in those cases where the source and the target of the representation *are* structures.

Chris Swoyer (1991, p. 452; see also Díez, 1998) for instance rightly claims that:

structural representation is not a necessary condition for representation in the ordinary sense of the word, since with sufficient perserverance—or perversity—we can use anything to represent virtually anything else, and in many cases the two things won't have any interesting structural similarities at all. And it is not sufficient for ordinary representation, since if you can find one structural representation of something, you can usually find many.

Swoyer is also precisely right in characterizing structural representation as having the “potential” to be used in surrogative reasoning about its target.

After having considered six different phenomenological constraints upon structural representation, Swoyer proposes the notion of a Δ/Ψ -morphism. (Swoyer's constraints implicitly rule out isomorphism, homomorphism and partial isomorphism as the relation of structural representation, thus adding grist to the mill of my critique of [iso] and its cousins.) Consider the representation of some structure B by means of another structure A; and consider two subsets of B's domain Δ and Ψ . Then Swoyer's notion is as follows:¹¹ a structure A *structurally represents* another structure B iff there is a (neither necessarily one-to-one nor onto) mapping $c: B \rightarrow A$ that preserves all the relations defined over Δ and counter-preserved all the relations defined over ψ , where ψ is non-empty. Since ψ is non-empty, structural representation serves always to carry out surrogate reasoning about its target. Swoyer's notion does not meet the logical, misrepresentation and non-sufficiency arguments presented here (in particular Δ/Ψ -morphisms are reflexive); neither is it meant to do so, since it is not meant as a theory of scientific representation in general. Yet Swoyer's work shows that [iso], [homo] and [partial iso] do not correctly describe even the means of *structural* representation!

7. Conclusions

One type of naturalism urges us to reduce the relation of scientific representation to facts about its relata. I have argued that no theory that attempts to reduce scientific representation to similarity or isomorphism will succeed. This might lead us to question the naturalism that gave rise to the need for reduction in the first place. There are other types of naturalism, which may offer different strategies to successfully naturalize scientific representation—but they will certainly not involve a reduction of representation to similarity or isomorphism.

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Notes

1. This does not diminish the strength of my argument. On the contrary, since I want to argue that there can be no radically naturalistic and substantive theory of the constituents of scientific representation, my argument would be weaker and less interesting the stronger the independent conditions on representation.
2. The inferential conception that I present in Suárez (forthcoming) sheds some light on the accuracy, reliability and explanatory power of representations.
3. I am not suggesting that Giere and van Fraassen have defended the conditions that I describe as [sim] and [iso] below; but they are often understood that way. This article can be seen as articulating clearly what Giere and van Fraassen can be taken to claim and what they cannot. I share with both an emphasis on the pragmatic dimension of representation: see van Fraassen (1992, 1994, 2000) and Giere (forthcoming).
4. Isomorphism is sometimes said to preserve, or amount to, “structural identity”. Such terminology is misleading since the isomorphic structures A and B are distinct: they have different objects in their domains. It is rather the “superstructure” of the logical properties of the relations in isomorphic structures that is identical. For that reason I prefer to use the phrase *identity of relational frameworks*.
5. In addition, questions of structure-exemplification are tricky. Which structure is exemplified by a concrete object is a highly context- and purpose-dependent issue. Consider, for instance, the many important structures that a bridge may exemplify besides geometric shape: the structure of weights and forces, the distribution of colours of each of the parts, the relative resistances of each part to air and water friction, etc. This underdetermination, or plurality, of structure seems to me a major objection to any form of structuralism; but it is somehow tangential to my concerns here.
6. Eric Peterson (unpublished manuscript) argues that the only claim that is essential to the semantic view is that theories are not linguistic entities. Nothing that I have written contradicts that minimal claim. One might agree that theories are better conceived as structures, while not agreeing that representation is a structural relation.
7. Blunt (1969), written in the midst of the cold war, probably overemphasizes the political aspects of *Guernica*. Chipp (1989), written during the controversy over *Guernica*'s return to the new Spanish democracy, and involved in the international diplomatic efforts that ensued, definitely underemphasizes them. The most balanced account may remain Arnheim (1962).
8. I employ *Guernica* to the same effect in Suárez (1999). French (forthcoming) misreports my argument as one of ambiguity between different targets, and then, confusingly, goes on to write in response that “it is not difficult to find other examples from the history of art which might be called non-representational” (p. 5). Ambiguity is no problem for [iso], since it is always possible for different objects to exemplify isomorphic structures. And I neither claim that *Guernica* is non-representational; that would be an absurd claim for me to make since it would bypass what is at stake, namely, whether there can be representation without isomorphism.
9. There does not seem to be a theory of mental intentionality that is free of problems, or has not already been refuted. Here is a sample list: similarity accounts of intentionality are refuted by, among others, Cummins (1989, Chap. 3) who also has strongly criticized covariance, or causal accounts (Cummins, 1989, Chaps 4–6, 1997). Egan (1998), MacDonald (1998) and Millikan (2000) offer strong and convincing arguments against Cummins's own isomorphism theory of mental representation; and Millikan's (1984) adaptive role theory, which is in any case not suited to explaining concrete instances of scientific representation, has been strongly criticized by ... Cummins (1997)!
10. This is not to say that representational forces cannot be studied scientifically. A good deal of historical and sociological research, for example, is in one way or another devoted to objectively settling issues of

past representational forces, and historians have developed some sophisticated tools to carry out these tasks: Baxandall (1985), for instance, was an influential milestone in the history of art. But although science can study values, it cannot reduce them to facts.

11. This is in fact Swoyer's "penultimate" definition. His final proposal includes an additional refinement to account for the further distinction between cases in which the representation correlates elements of B uniquely to elements of A and those in which the representation correlates elements of A uniquely to elements of B. Since the distinction is only required to cover cases of linguistic, or word-to-object representation, I ignore it here.

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